**Mathematical Economics Notes \_III for V sem**

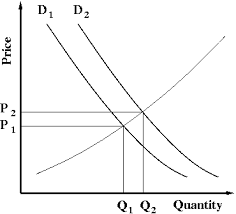
**A. Diagram discussion Economic Relationship**

**Interpretation**

**1. Relation\* Price and quantity demanded**

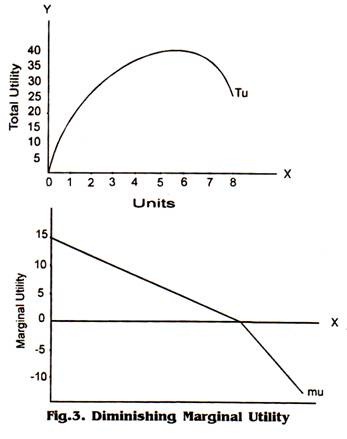
**2. Relation between interest rate and volume of investment with interest rate on y axis and volume of investment on x axis.**

**A.2.i**

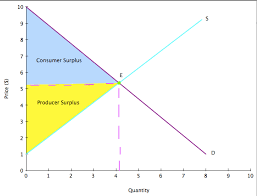


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**A.2.ii.**



**A.2.iii Consumer surplus**



**B.1 Matrix application**

As finding of equilibrium solution is fundamental to the study of Economic problems, Matrix is widely used in Economics. In utility analysis, theory of firm input-output analysis and many other areas, the concept is widely used.

Applications of System of Linear Equations

Solution of Equation with the help of Determinants – Cramer’s Rule

Let the system of equations be

a1x + b1y+c1z =m1

a2x + b2y + c2z = m2

a3x +b3y +c3z = m3

Example 1.

x-2 y+ 3z = 1

3x-1y+ 4z =3

2x+ y-2z = -1

Then the solution is given by Cramer’s Rule

1 x y z

---------------------------- = ----------------------------- = --------------------------- = ---------------------------

1 -2 3 1 -2 3 1 1 3 1 -2 1

3 -1 4 3 -1 4 3 3 4 3 -1 3

2 1 -2 -1 1 -2 2 -1 -2 2 1 -1

1 x y z

or, ---------------------------- = ----------------------------- = --------------------------- = ---------------------- - 30+15 -6+6 -29+14 -20+5

1 x y z

or, ---------------------------- = ----------------------------- = --------------------------- = --------------------

-15 0 -15 -15

or, x=0/-15= 0, y= -15/-15 = 1; z = -15/-15 = 1

Example2 – A Firm which produces three products X, Y and Z requires the mix of three inputs A, B and C as below:

Firm = A B c

X 3 4 1

Y 2 3 4

Z 1 2 1

Using matrix algebra, find

i. the total requirements of each material if the firm produces 200 units of each product

ii. The price/unit of each input A, B, and C is Rs. 4, Rs. 7and Rs. 8 respectively. What will be total cost.

Solution-

X V Z A B c

200 200 200 3 4 1

2 3 4

1 2 1

Requirement of A = 3x200+2x200+1x200 = 600+400+200 = 1200 unitsx Rs. 4 = Rs. 4800

Requirement of B = 4x200+3x200+2x200 = 800+600+400 = 1800 unitsx Rs. 7 = Rs.1,2600

Requirement of C = 1x200+4x200+1x200 = 200+800+200 = 1200 unitsx Rs. 8 = Rs. 9600

Example3 – If supply and demand function of a firm are:

2p + 2x= 27 (Demand)

6p-2x = 9 (Supply)

Find the equilibrium price and supply. If a tax of Rs. 3/2 per unit of a commodity is imposed, find the new equilibrium

**C. Differential Calculus**

C.1 Profit Maximisation under various market Conditions

In Under Perfect competition MR = AR and the difference between MR and AR gives Monopoly power of the firm.

**D. Integral Calculus**

**D.1 Profit maximisation under different Market conditions**

MR is constant and equal to AR. MR/AR is equal to x axis. It is same for all firms

Question 1 The marginal cost and marginal revenue of a firm are given as

MC = 6 +0.12q and MR = 18

Compute the total profit, given the total cost is zero when there is NIL output.

Solution

For Profit Maximization

MC = MR

Or, 6 +0.12q = 18

Or, 0.12 q = 18- 6 or, q=12/0.12 =100

MR is CONSTANT

P=AR=MR= 18

So, Total Revenue = 18q

TC = ∫(6+0.12q)dq = 6q+0.12q2/2+ A

Π (profit)= TR- TC = 18q-(6q+0.06q2) = 12q - 0.12q2

= 12x100-10000x0.06 = 1200- 6000 = Rs. 600

**D.2 Capital Formation**

Capital formation is the process of addition to a stock of capital.

Rate of capital formation may thus be expressed as dk/dt

Where k is a function of time t. K(t) = ∫(dk/dt). dt

**D.3 Compound Interest & Capital Value and Flow value**

Let us suppose that we obtain different income in different years from an investment

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Years | This year | 1st year | 2nd Year | 3rd Year | ----- | Mth year |
| Income in Rs. | a0 | a1 | a2 | a3 | ----- | am |

So the the present value of a1, a2, a3 and so on am if the rate of interest is r% per year is the CAPITAL VALUE of the present investment.

If interest is added n times within a year than we multiply the time by n and again, divide the rate of interest by n.

But if it continuously added and x is the number of year for which this income is received, we have,

Y = ∫0x ae-rt dt = [-a/r.e-rt ]0x

= a/r(1-e-rx)= Capitalise value of the continuous stream

So, the capitalised value of continuous equal stream depends on the size of income stream, number of years it flows and the rate of interest. This is called FLOW VALUE.

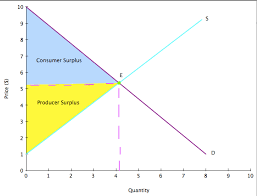
Example- If the interest is continuously added at 12% per year, what will be the capital value of uniform income stream of Rs. 100 per year for 10 years (e= 2.71828).

Solution- Capital value : y = a/r(1-e-rt)

Here, a= 100; r =12/100 =0.12 and Time t= 10 years

So, y = (100/ 0.12) (1-e-0.12x10) = 833.33 {1-(2.71828)-1.2}

**D.4. Consumer’s Surplus under Pure competition and monopoly and Producer’s Surplus**



Consumer’s Surplus is the difference between what the consumer is ready to pay rather than going without it minus what he actually pays = M-N

Where,

M = What the consumer’s is ready to pay = Total area below the demand function from 0 to X0 (which can be calculated under the curve)

N- price at which the quantity sold x quantity sold= pxq

Producer’s Surplus is the difference between a producer earn by selling a particular quantity at the given minus the price at which he would like to sell rather than going without selling

PS = N-R

Where R= Producers will like to sell rather than going without selling it

Significance of understanding Consumer surplus for producer and producer Surplus for consumer

Exercise - The demand function for a commodity is p= 24-3q, and P=3q is the supply function. Find the consumer surplus.

Solution- p= 24-3q; TR= pq = (24-3q)q = 24q-3q2 =

Supply function, p=3q So, pq= 3q2

Demand Function = Supply Function

Or, 24q-3q2 = 3q2

Or, d/dq (24q-3q2-3q2)=0

Or, 24-6q=0 ; or, q =4 and p= 12

So, Consumer Surplus = ∫40 (24-3q )dq - pq= [24q-3q2/2]40 – 4x12

=24x4 -24- 48 = 96 – 72 = 24

**E APPLICATION OF DIFFERENTIAL EQUATION**

**E.1 Market Price Function**

Suppose the demand and supply function: when p is the price, Qd is the quantity demanded and Qs is the quantity supplied are:

Qd = a- bp (a, b >0)----------- (i)

Qs = -c+ dp (c, d>0) ----------- (ii)

and dp/dt =α (Qd-Qs) (α >0) ---------- (iii)

[ Equation (3) implies that changes that change in price with respect to time (t) is directly proportional to the excess of demand over supply (=Qd-Qs)]

Subtracting (i) and (ii) in Equation (iii), we get

dp/dt = α[(a-bp) –(-c+dp)]

dp/dt = α [(a+c) –(b+d)p] --------------------------(iv)

equation (i) and (ii) give the equilibrium price pn:

(a+c) = (b+d) pn --------------------------------------(v)

Substituting equat (5) in (4), we get

dp/dt = α [(b+d)pn – (b+d)p]

dp/dt = α (b+d)(pn – p)

dp/dt + α (b+d)p = α (b+d)pn -------------------- [where k = α (b+d)]

**E.2 Dynamic Multiplier**

The concept of employment multiplier was first introduced in Economic Theory by Prof. R.F. Kahn, a Cambridge economist, in his article entitled, “The Relation of Home Investment to Unemployment” . The theory was further developed by Keynes when he discussed investment multiplier. The investment multiplier explains the cumulative effects of changes in investment on income via their effects on consumption expenditure.

△ Y = △I.k

Where , △ Y = Change of National Income

△I=Change in Investment

K = Value of Multipler

**Relation between Consumption and Income**

C= α +β Y

Where C= consumption

α = intercept

β = slope of the curve

As income increases, consumption also increases but the slop of consumption curve than the Income curve.

**E.3 Marginal Propensity to Consume (MPC)-**

**MPC** is the proportion of increase in consumption with additional unit of increase in income. For example, if a household earns one extra dollar of disposable income, and the marginal propensity to consume is 0.65, then of that dollar, the household will spend 65 cents and save 35 cents.

MPC= dc/dY

Where dc= increase in consumption

dY = increase in additional unit of Household income (in Economics, Household is smallest unit. All those who eat from same kitchen are member of same household)

MPS = 1- MPC

Where MPS = Marginal Propensity to save

So, MRS+MPC = 1